Radiation of Closed Strings between the Parallel Dynamical-Dressed Unstable Dp-Branes

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Abstract

We introduce a boundary state which is corresponding to a Dp-brane with tangential dynamics in the presence of the Kalb-Ramond field, a tachyonic field and a \( U(1) \) gauge potential in a special gauge. From the interaction of such branes radiation amplitude of a general massless closed string will be computed. The effects of the large distances of the branes on this radiation will be studied. In the large distances of the branes, the possibility of axion radiation will be investigated. Our calculations will be in the context of the bosonic string theory.

PACS numbers: 11.25.-w; 11.25.Uv

Keywords: Boundary state; Background fields; Dynamics; Radiation amplitude; Axion production.
1 Introduction

By studying the D-branes properties and their interactions, some of the important physical results in the string theory were obtained [1, 2]. One of the convenient methods for calculating the interaction of two branes, in the closed-channel, is the boundary state formalism [3]-[15]. The interaction of two bare-static D-branes in the superstring theory is zero [16]. By adding dynamics and various background fields, some nonzero and valuable interaction amplitudes were obtained [17]-[27].

Closed strings can be radiated from the Dp-branes with various configurations. One of the most important setups is the radiation of closed strings from a single unstable Dp-brane [28, 29, 30]. Besides the tachyonic background, which prominently induces instability on the branes, closed string radiation from a single unstable brane in the presence of the various background fields has been also investigated [31, 32]. In addition, the supersymmetric version of the closed string radiation was also constructed [33]. Another interesting configuration is the closed string radiation from the interacting D-branes. This kind of radiation was studied only in specific setups [34, 35]. We shall add dynamics, internal and background fields to the interacting unstable branes to extract the closed string radiation from a generalized configuration.

The background fields and dynamics of the branes motivated us to obtain the effects of these variables on the closed string radiation from the interacting branes. Therefore, in this paper, in the context of the bosonic string theory, we investigate the massless closed string radiation from the interaction of two unstable Dp-branes with tangential dynamics in the presence of the Kalb-Ramond field, a quadratic tachyonic field and a U(1) gauge potential in a special gauge. For computing the radiation amplitude, we shall apply the boundary state formalism. Thus, by inserting an appropriate vertex operator in the worldsheet of the exchanged closed string between the branes, we produce a radiated closed string. We accurately use the eikonal approximation in which the recoil of the branes is ignored. After calculations for radiating a general massless state, we concentrate on the axion radiation from the distant branes. We shall observe that the radiated axion is directly emitted by one of the two interacting branes. Note that the interaction between
two D-branes in the large distance limit occurs only via the exchange of the massless states.

This paper is organized as follows. In Sec. 2, the boundary state, corresponding to a dynamical-dressed unstable Dp-brane, will be introduced. In Sec. 3.1, the radiation amplitude of a general massless closed string from the interaction of two parallel Dp-branes will be constructed. In Sec. 3.3, the foregoing radiation amplitude will be deformed for the distant branes. In Sec. 4, the radiation of an axion from the distant branes will be obtained. Sec. 5 is devoted to the conclusions.

2 The boundary state of the dynamical-dressed unstable Dp-brane

In this section we introduce the boundary state corresponding to a dynamical-dressed unstable Dp-brane. Hence, we start with the following sigma-model action for closed string

$$S = - \frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma \left( \sqrt{-\text{det} g} g^{\alpha\beta} G_{\mu\nu} \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu} + \varepsilon^{\alpha\beta} B_{\mu\nu} \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu} \right) + \frac{1}{2\pi\alpha'} \int_{\partial\Sigma} d\sigma \left( A_\alpha \partial_\sigma X^{\alpha} + \omega_{\alpha\beta} J^{\alpha\beta}_\tau + T(X^{\alpha}) \right),$$

where $\mu, \nu \in \{0, 1, \cdots, 25\}$ represent the spacetime indices, which are split into the indices for the worldvolume directions, i.e. $\alpha, \beta \in \{0, \cdots, p\}$, and for the perpendicular directions to it, i.e. $i, j \in \{p+1, \cdots, 25\}$. Besides, $a, b \in \{0, 1\}$ specify the worldsheet directions. $\Sigma$ is the closed string worldsheet, while $\partial\Sigma$ is its boundary. $G_{\mu\nu}$ and $g_{ab}$ are the metrics of the target spacetime and string worldsheet, respectively. The background fields are the Kalb-Ramond field $B_{\mu\nu}$, the U(1) internal gauge field $A_\alpha$ and the open string tachyon field $T(X^{\alpha})$. The constant antisymmetric tensor $\omega_{\alpha\beta}$ shows the angular velocity of the brane. Hence, the $\omega$-term obviously expresses the tangential dynamics of the brane, whose explicit form is given by $\omega_{\alpha\beta} J^{\alpha\beta}_\tau = 2\omega_{\alpha\beta} X^{\alpha} \partial_\tau X^\beta$. Besides the tangential dynamics, one can also impose a transverse dynamics to the brane. This can be solely exerted by the boost transformations on the boundary state equations [22], [32], [33], [36], [37], [38]. However, for simplicity, we shall consider only the tangential motion and rotation.
We apply the flat spacetime with the metric $G_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1,+1,\cdots,+1)$ and a constant Kalb-Ramond field $B_{\mu\nu}$. Besides, we utilize the trusty gauge $A_{\alpha} = -\frac{1}{2} F_{\alpha\beta} X^\beta$ with the constant field strength $F_{\alpha\beta}$. In addition, we use the quadratic tachyon profile $T = \frac{1}{2} U_{\alpha\beta} X^\alpha X^\beta$ where $U_{\alpha\beta}$ is a constant symmetric matrix. We should note that the origin and the conformal invariance of the action (2.1) have been widely studied in various papers, e.g., see the Refs. [3], [39], [40], [41], [42], [43] and [44] (and also references therein).

By setting the variation of the action to zero we receive the equation of motion and the following boundary state equations

$$\left[ A_{\alpha\beta} \partial_\tau X^\beta + \mathcal{F}_{\alpha\beta} \partial_\sigma X^\beta + U_{\alpha\beta} X^\beta \right]_{\tau=0} |B\rangle = 0,$$

$$\delta X^i|_{\tau=0} |B\rangle = 0,$$

(2.2)

where the total field strength is $\mathcal{F}_{\alpha\beta} = F_{\alpha\beta} - B_{\alpha\beta}$ and $A_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta}$. For the next purpose we bring in the solution of the equation of motion of $X^\mu(\sigma, \tau)$,

$$X^\mu(\sigma, \tau) = x^\mu + 2\alpha' Q^\mu T + \frac{i}{2} \sqrt{2}\alpha' \sum_{m\neq 0} \frac{1}{m} \left( \alpha^\mu_m e^{-2im(\tau-\sigma)} + \bar{\alpha}^\mu_m e^{-2im(\tau+\sigma)} \right).$$

(2.3)

In fact, because of the presence of the background fields on the brane worldvolume, the Lorentz symmetry has been clearly broken. Thus, the tangential dynamics of the brane along its worldvolume directions is sensible. Now we prove this. The effect of the Lorentz generators on the boundary state can be obtained from Eq. (2.2),

$$J^{\alpha\beta} |B\rangle = \int_0^\pi d\sigma \left[ (A^{-1}\mathcal{F})^\alpha_\gamma X^\beta_\sigma X^\gamma - (A^{-1}\mathcal{F})^\beta_\gamma X^\alpha_\sigma X^\gamma \right]_{\tau=0} |B\rangle.$$

(2.4)

This equation prominently demonstrates that for restoring the Lorentz symmetry, the tachyon matrix $U_{\alpha\beta}$ and the total field strength $\mathcal{F}_{\alpha\beta}$ should vanish. We see that even in the absence of the electric and magnetic fields, the tachyon field independently breaks the Lorentz symmetry along the worldvolume of the brane. However, since the RHS of Eq. (2.4) depends on the spacetime coordinates along the brane worldvolume we conclude that the Lorentz symmetry breaking is local. Therefore, the internal linear motion and rotation of the brane in any direction clearly are sensible.
For receiving more perception of the tangential dynamics of a Dp-brane, beside the Lorentz symmetry breaking, we should note that such setups are T-dual of some imaginable systems. In other words, a Dp-brane with the tangential dynamics can be constructed via T-duality from a D(p − 1)-brane which moves and rotates parallel and perpendicular to its volume. For example, consider a D-string along the direction $x^1$ with the velocity components $V^1$ and $V^2$ in the $x^1$- and $x^2$-directions, respectively. Now apply the T-duality in the direction $x^2$, in which we suppose that it is compact. The resultant system is a D2-brane, which has been expanded along the plane $x^1x^2$ with the velocity $V^1$ in the direction $x^1$ and an electric field $E = V^2$ in the direction $x^2$. In the same way, one can acquire a rotating D2-brane through the T-duality from another setup of a D-brane. Note that after exerting the T-duality all compact coordinates can be decompactified.

Combining Eqs. (2.2) and (2.3), the boundary state equations are conveniently expressed in terms of the zero modes and oscillators. The coherent state method enables us to obtain the solution of the oscillatory part of the boundary state equations

$$ |B\rangle^{(\text{osc})} = \prod_{n=1}^{\infty} [\det Q(n)]^{-1} \exp \left[ -\sum_{m=1}^{\infty} \frac{1}{m} (\alpha_{\mu}^{\alpha} S_{(m)\mu\nu} \tilde{\alpha}_{\nu}^{\alpha} - m) \right] |0\rangle_{\alpha} \otimes |0\rangle_{\tilde{\alpha}}, \tag{2.5} $$

where the normalization factor $\prod_{n=1}^{\infty} [\det Q(n)]^{-1}$ comes from the disk partition function. In fact, the advent of this prefactor is due to the square structure of the action (2.1). That is, in addition to the gauge choice $A_{\alpha} = -\frac{1}{2} F_{\alpha\beta} X^{\beta}$, the other terms in the action also possess the squared form of $X^{\alpha}$s. Thus, this configuration path-integrally is feasible and the normalizing prefactor must be inserted [11]. The matrices possess the following definitions

$$ Q_{(m)\alpha\beta} = A_{\alpha\beta} - F_{\alpha\beta} \pm \frac{i}{2m} U_{\alpha\beta}, \tag{2.6} $$

$$ S_{(m)\mu\nu} = (\Delta_{(m)\alpha\beta}, -\delta_{ij}), \tag{2.7} $$

$$ \Delta_{(m)\alpha\beta} = (Q_{(m)}^{-1} N_{(m)})_{\alpha\beta}, \tag{2.8} $$

$$ N_{(m)\alpha\beta} = A_{\alpha\beta} + F_{\alpha\beta} \mp \frac{i}{2m} U_{\alpha\beta}, \tag{2.9} $$
For the zero-mode part of the boundary state we receive the following expression

\[ |B\rangle^{(0)} = \prod_i \delta(x^i - y^i)|p^i = 0\rangle \int_{-\infty}^{\infty} dp^\alpha \exp \left[ i\alpha' \left( (U^{-1}A)_{\alpha\alpha}(p^\alpha)^2 \right. \right. \\
+ \left. \left. \sum_{\beta \neq \alpha} (U^{-1}A + A^T U^{-1})_{\alpha\beta}(p^\alpha p^\beta) \right) \right] \bigg| p^\alpha \bigg\rangle. \] (2.10)

By considering the contribution of the conformal ghosts to the boundary state, which is

\[ |B\rangle^{(gh)} = \exp \left[ \sum_{m=1}^{\infty} \left( c_{-m} \tilde{b}_{-m} - b_{-m} \tilde{c}_{-m} \right) \right] \frac{c_0 + \tilde{c}_0}{2} |q = 1\rangle \otimes |\tilde{q} = 1\rangle, \] (2.11)

one can construct the total boundary state

\[ |B\rangle^{(tot)} = \frac{T_p}{2} |B\rangle^{(osc)} \otimes |B\rangle^{(0)} \otimes |B\rangle^{(gh)}, \] (2.12)

where \( T_p \) is the Dp-brane tension.

3 Radiation of a general massless closed string

In this section we calculate the radiation amplitude of a general massless closed string from the interaction of two parallel dynamical-dressed unstable Dp-branes. To generalize our calculations let us assume that the fields and dynamics of the two interacting branes to be different. Therefore, the subscripts (1) and (2) will be used to exhibit these differences.

3.1 The radiation amplitude

In the closed string channel, the interaction of two D-branes takes place by exchanging a closed string between the branes. The geometry of the exchanged string worldsheet is a cylinder with \( \tau \) as the coordinate along the length of the cylinder, \( 0 \leq \tau \leq t \), and \( \sigma \) as the periodic coordinate, i.e. \( 0 \leq \sigma \leq \pi \). The radiation of a closed string state is elaborated by inserting the corresponding vertex operator of the string state, i.e. \( V(\tau, \sigma) \), into the amplitude of the interaction. Precisely, one should compute

\[ A = \int_0^\infty dt \int_0^t d\tau \langle B_1|e^{-tH^{(tot)}}V(\tau, \sigma)|B_2\rangle^{(tot)}, \] (3.1)
where $H_{(\text{tot})}$ comprises the ghost and matter parts of the closed string Hamiltonian

$$H_{(\text{tot})} = H_{\text{ghost}} + \alpha' Q^2 + 2 \sum_{n=1}^{\infty} (\alpha_{-n} \cdot \alpha_{n} + \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_{n}) - 4. \quad (3.2)$$

Let us apply $z = \sigma + i\tau$ and $\partial = \partial_2$. Subsequently, the vertex operator for a general massless string possesses the feature

$$V(z, \bar{z}) = \epsilon_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu e^{ip \cdot X}, \quad (3.3)$$

where $\epsilon_{\mu\nu}$ is the polarization tensor and $p^\mu$ (with $p^\mu p_\mu = 0$) is the momentum of the radiated closed string.

Since each string coordinate is the summation of the zero-mode part and oscillating portion, i.e. $X^\mu(\tau, \sigma) = X_0^\mu(\tau) + X_{\text{osc}}^\mu(\sigma, \tau)$, we can write

$$\partial X^\mu \bar{\partial} X^\nu e^{ip \cdot X} = \left( \partial X_0^\mu \bar{\partial} X_0^\nu + \partial X_0^\mu \bar{\partial} X_{\text{osc}}^\nu + \partial X_{\text{osc}}^\mu \bar{\partial} X_0^\nu + \partial X_{\text{osc}}^\mu \bar{\partial} X_{\text{osc}}^\nu \right) e^{ip \cdot X_0} e^{ip \cdot X_{\text{osc}}}. \quad (3.4)$$

This implies that the appearing terms in the vertex operator can be written in the general form

$$\epsilon_{\mu\nu} \left[ \Gamma(X_0) \Lambda(X_{\text{osc}}) \right]^{\mu\nu} e^{ip \cdot X_0} e^{ip \cdot X_{\text{osc}}}, \quad (3.5)$$

where $\Gamma(X_0) \in \{1, \partial X_0, \bar{\partial} X_0, \partial X_0 \bar{\partial} X_0\}$ and $\Lambda(X_{\text{osc}}) \in \{1, \partial X_{\text{osc}}, \bar{\partial} X_{\text{osc}}, \partial X_{\text{osc}} \bar{\partial} X_{\text{osc}}\}$.

This notation allows us to separate the computation as follows

$$(\text{tot}) \langle B_1| e^{-tH_{\text{tot}}} V(\tau, \sigma)|B_2 \rangle^{(\text{tot})} = \frac{T_p^2}{4} \epsilon_{\mu\nu} \left[ \langle 0 | e^{-tH(X_0)} \Gamma(X_0) e^{ip \cdot X_0} |B_2 \rangle^{(0)} \times \langle \text{osc} | e^{-tH_{\text{osc}}} \Lambda(X_{\text{osc}}) e^{ip \cdot X_{\text{osc}}} |B_2 \rangle^{(\text{osc})} \right]^{\mu\nu} \times \langle g | B_1| e^{-tH(g)} |B_2 \rangle^{(g)}, \quad (3.6)$$

where $H(X_0) = \alpha' Q^2 - 4$. 

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After very long calculations, for the various forms of $\Gamma(X_0)$, we receive

$$
\langle B_1 | e^{-tH(X_0)} e^{ip \cdot X_0} | B_2 \rangle = (2\pi)^{26} \mathcal{D}(y_1, y_2) e^{4t} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \prod_{\alpha=0}^{p} dk^{\alpha} dk'^{\alpha} \times \mathcal{D}(k; k') e^{-\alpha' (l' k^{2} + \tau k'^{2})},
$$

$$
\langle B_1 | e^{-tH(X_0)} \partial X_\mu^{(0)} e^{ip \cdot X_0} | B_2 \rangle = -\alpha' (2\pi)^{26} \mathcal{D}(y_1, y_2) e^{4t} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \prod_{\alpha=0}^{p} dk^{\alpha} dk'^{\alpha} \times \mathcal{D}(k; k') k^{\mu} e^{-\alpha' (l' k^{2} + \tau k'^{2})},
$$

$$
\langle B_1 | e^{-tH(X_0)} \partial X_\mu^{(0)} \partial X_\nu^{(0)} e^{ip \cdot X_0} | B_2 \rangle = \alpha' (2\pi)^{26} \mathcal{D}(y_1, y_2) e^{4t} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \prod_{\alpha=0}^{p} dk^{\alpha} dk'^{\alpha} \times \mathcal{D}(k; k') k^{\mu} k'^{\nu} e^{-\alpha' (l' k^{2} + \tau k'^{2})},
$$

(3.7)

where, by applying the Wick's rotation $\tau \rightarrow -i\tau$, we introduced another proper time $l' = t - \tau$. Additionally, the functions $\mathcal{D}(y_1, y_2)$ and $\mathcal{D}(k; k')$ have the following definitions

$$
\mathcal{D}(y_1, y_2) = \prod_{i=p+1}^{25} \delta(x^i - y_1^i) \delta(x^i - y_2^i) \delta(p^i),
$$

(3.8)

$$
\mathcal{D}(k; k') = \prod_{\alpha=0}^{p} \delta (p^{\alpha} + k'^{\alpha} - k^{\alpha})
\times \exp \left[ -i \alpha' \left( \sum_{\alpha=0}^{p} \left[ (U_1^{-1} A_1)_{\alpha \alpha} (k^{\alpha})^2 - (U_2^{-1} A_2)_{\alpha \alpha} (k'^{\alpha})^2 \right] + 2 \sum_{\alpha \neq \beta} \left[ (U_1^{-1} A_1)_{\alpha \beta} k^{\alpha} k^{\beta} - (U_2^{-1} A_2)_{\alpha \beta} k'^{\alpha} k'^{\beta} \right] \right) \right].
$$

(3.9)

In Eq. (3.6), for the factor which includes the oscillators, we write

$$
\langle B_1 | e^{-tH(\text{osc})} \Lambda(X_{\text{osc}}) e^{ip \cdot X_{\text{osc}}} | B_2 \rangle^{(\text{osc})} \equiv \langle \Lambda(X_{\text{osc}}) e^{ip \cdot X_{\text{osc}}} \rangle Z^{(\text{osc})},
$$

(3.10)

where $Z^{(\text{osc})}$ is the oscillation part of the partition function

$$
Z^{(\text{osc})} = \langle B_1 | e^{-tH(\text{osc})} | B_2 \rangle^{(\text{osc})}.
$$

(3.11)

The other factor in Eq. (3.10) is given by

$$
\langle \Lambda(X_{\text{osc}}) e^{ip \cdot X_{\text{osc}}} \rangle \equiv \frac{\langle B_1 | e^{-tH(\text{osc})} \Lambda(X_{\text{osc}}) e^{ip \cdot X_{\text{osc}}} | B_2 \rangle^{(\text{osc})}}{\langle B_1 | e^{-tH(\text{osc})} | B_2 \rangle^{(\text{osc})}}.
$$

(3.12)
For the various forms of $\Lambda(X_{osc})$ Eq. (3.12) finds the following features
\[
\langle \partial X^\mu e^{ip\cdot X_{osc}} \rangle = i \langle \partial X^\mu p \cdot X \rangle_{osc} \langle e^{ip\cdot X_{osc}} \rangle, \tag{3.13}
\]

\[
\langle \bar{\partial} X^\mu e^{ip\cdot X_{osc}} \rangle = i \langle \bar{\partial} X^\mu p \cdot X \rangle_{osc} \langle e^{ip\cdot X_{osc}} \rangle, \tag{3.14}
\]

\[
\langle \partial X^\mu \bar{\partial} X^\nu e^{ip\cdot X_{osc}} \rangle = \left[ \langle \partial X^\mu \bar{\partial} X^\nu \rangle_{osc} - \langle \partial X^\mu p \cdot X \rangle_{osc} \langle \bar{\partial} X^\nu p \cdot X \rangle_{osc} \right] \langle e^{ip\cdot X_{osc}} \rangle. \tag{3.15}
\]

The new proper time $l' = t - \tau$ enables us to change the integrations as
\[
\int_0^\infty dt \int_0^t d\tau = \int_0^\infty d\tau \int_0^\infty dl'. \tag{3.16}
\]

In fact, $l'$ and $\tau$ indicate the proper times of the radiated closed string from the right (first) brane and the left (second) brane, respectively. Therefore, $\tau = 0$ ($l' = 0$) specifies the radiation from the first (the second) brane, i.e. that brane with the boundary state $|B_1\rangle$ ($|B_2\rangle$). The case $\tau, l' > 0$ obviously corresponds to radiation that has been occurred between the branes.

Adding all these together we can write the general radiation amplitude as
\[
\mathcal{A} = T^2_p \frac{(2\pi)^{26}}{4} \int_0^\infty d\tau \int_0^\infty dl' \mathcal{D}(y_1, y_2) e^{\mathcal{A}(l'+\tau)} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \prod_{\alpha=0}^{p} dk^\alpha dk'^\alpha \mathcal{D}(k; k') \times e^{-\alpha'(l'k^2+\tau k'^2)} Z_{osc, g} \langle e^{ip\cdot X_{osc}} \rangle \mathcal{M}, \tag{3.17}
\]

where $\mathcal{M}$ and $Z_{osc, g}$ are given by
\[
\mathcal{M} = \epsilon_{\mu\nu} \left\{ \langle \partial X^\mu \bar{\partial} X^\nu \rangle_{osc} - \langle \partial X^\mu p \cdot X \rangle_{osc} \langle \bar{\partial} X^\nu p \cdot X \rangle_{osc} - i\alpha' k^\mu \langle \bar{\partial} X^\nu p \cdot X \rangle_{osc} \right. \\
left. + i\alpha' k'^\nu \langle \partial X^\mu p \cdot X \rangle_{osc} - \alpha'^2 k^\mu k'^\nu \right\}, \tag{3.18}
\]

\[
Z_{osc, g} = \langle osc | B_1 | e^{-\left(\tau+\tau'\right)H_{osc}} | B_2 \rangle_{osc} (g) \langle B_1 | e^{-\left(\tau+\tau'\right)H_{osc}} | B_2 \rangle_{osc}. \tag{3.19}
\]

According to the factor $e^{-\alpha'(l'k^2+\tau k'^2)}$ the integration over $l'$ and $\tau$ leads to the factors similar to $1/k^2$ or $1/k'^2$ or both, which correspond to the propagators of the radiated massless strings by the branes.

### 3.2 The correlators

Now we should compute all correlators in Eq. (3.17). In Eqs. (3.7) we employed the Wick’s rotation to determine the zero-mode correlators. Now we should extract the
oscillatory correlators in this frame. Thus, after some heavy calculations we acquire

\[ Z^{(\text{osc},g)} = \prod_{n=1}^{\infty} \det \left[ Q_{(n)1}^\dagger Q_{(n)2} \right]^{-1} \prod_{n=1}^{\infty} \frac{(1 - q^{2n})^2}{1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}} , \]  

(3.20)

\[
\langle \partial X\mu X^{\nu} \rangle_{\text{osc}} = i\alpha' \sum_{n=1}^{\infty} \left\{ \eta^{\mu\nu} \text{Tr} \left( \frac{1 + S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}}{1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}} \right) - S_{(n)}^{(1)\mu} S_{(n)}^{(2)\nu} \text{Tr} \left( \frac{S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}}{1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}} \right) - S_{(n)}^{(2)\mu} \text{Tr} \left[ \frac{S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2(n-1)} e^{-4\tau}}{1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2(n-1)} e^{-4\tau}} \right] \right. \\
\times \left( 1 + \frac{1}{1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2(n-1)} e^{-4\tau}} \right) \right. \\
\left. + S_{(n)}^{(1)\mu} \text{Tr} \left[ \frac{S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2(n-1)} e^{-4\tau}}{1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2(n-1)} e^{-4\tau}} \right] \right) \right\} , \]  

(3.21)

\[
\langle \partial X\mu \tilde{\partial} X^{\nu} \rangle_{\text{osc}} = 2\alpha' \sum_{n=1}^{\infty} \left\{ 2 n \eta^{\mu\nu} \text{Tr} \left( \frac{S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}}{1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}} \right) - n S_{(n)}^{(1)\mu} S_{(n)}^{(2)\nu} \text{Tr} \left( \frac{S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}}{1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}} \right) + 2 n S_{(n)}^{(2)\mu} \text{Tr} \left( \frac{S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2(n-1)} e^{-4\tau}}{1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2(n-1)} e^{-4\tau}} \right)^{\frac{1}{3}} \right) \\
+ 2 n S_{(n)}^{(2)\mu} \text{Tr} \left( \frac{S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2(n-1)} e^{-4\tau}}{1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2(n-1)} e^{-4\tau}} \right)^{\frac{1}{3}} \right) \right\} , \]  

(3.22)

where \( q = e^{-2(\tau + \ell)} \). One can conveniently show that \( \langle \partial X\mu X^{\nu} \rangle_{\text{osc}} = -\langle \tilde{\partial} X\mu X^{\nu} \rangle_{\text{osc}} \).

Using the Cumulant expansion, the exponential \( e^{ip\cdot X_{\text{osc}}} \) can be elaborated in terms of the correlators of \( X \)'s. According to the boundary states formulation and Eq. (3.12),
the correlators with odd number of \( X \)'s vanish. Hence, only the correlators with even number of \( X \)'s remain. Now let us assume that the momentum of the radiated string is small. This implies that in the Cumulant expansion one should only compute the factor \( \exp \left( - \frac{1}{2} p_\mu p_\nu \langle X^\mu X^\nu \rangle_{osc} \right) \). Subsequently, for the massless radiated strings, the following result is received

\[
\langle e^{ip \cdot X_{osc}} \rangle = \prod_{n=1}^{\infty} \left\{ \det \left( 1 - S^{(1)\dagger}_n S^{(2)}_n q^{2n} \right) \frac{\alpha'}{2\pi p_\mu p_\nu S^{(2)\mu\nu}_n} \right\} \times \det \left( 1 - S^{(1)\dagger}_n S^{(2)}_n q^{2(n-1)} e^{-4\tau} \right) \frac{\alpha'}{2\pi p_\mu p_\nu S^{(1)\mu\nu}_n} \times \det \left[ \exp \left( \frac{\alpha' p_\mu p_\nu S^{(2)\mu\nu}_n}{2n} \left( 1 - S^{(1)\dagger}_n S^{(2)}_n q^{2(n-1)} e^{-4\tau} \right)^{-1} \right) \right] \times \det \left[ \exp \left( \frac{\alpha' p_\mu p_\nu S^{(1)\mu\nu}_n}{2n} \left( 1 - S^{(1)\dagger}_n S^{(2)}_n q^{2(n-1)} e^{-4\tau} \right)^{-1} \right) \right] \right\}. 
\]

(3.23)

### 3.3 The large distance branes

Here we consider radiation of the closed strings from the branes which are located far from each other. In this case only the exchange of the massless closed strings possesses the dominant contribution to the interaction. Since the long enough time corresponds to the large distance of the branes, the limit \( t \to \infty \) should merely exert on the oscillating part of the amplitude, i.e. on Eqs. (3.20)-(3.23). Accordingly, \( Z^{(osc,g)} \) and the correlators in the amplitude (3.17) find the following forms

\[
Z^{(osc,g)}|_{t\to\infty} = \prod_{n=1}^{\infty} \det \left[ Q^{(n)\dagger}_1 Q^{(n)\dagger}_2 \right]^{-1},
\]

(3.24)

\[
\langle \partial X^\mu X^\nu \rangle_{osc}|_{t\to\infty} = i\alpha' \left\{ -13 \eta^{\mu\nu} - S^{(2)\mu\nu}_1 \text{Tr} \left[ \frac{S^{(1)\dagger}_1 S^{(2)}_1 e^{-4\tau}}{1 - S^{(1)\dagger}_1 S^{(2)}_1 e^{-4\tau}} \left( 1 + \frac{1}{1 - S^{(1)\dagger}_1 S^{(2)}_1 e^{-4\tau}} \right) \right] \right. 

+ \left. S^{(1)\mu\nu}_1 \text{Tr} \left[ \frac{S^{(1)\dagger}_1 S^{(2)}_1 e^{-4\tau}}{1 - S^{(1)\dagger}_1 S^{(2)}_1 e^{-4\tau}} \left( 1 + \frac{1}{1 - S^{(1)\dagger}_1 S^{(2)}_1 e^{-4\tau}} \right) \right] \right\},
\]

(3.25)
\[
\langle \partial X^\mu \partial X^\nu \rangle_{osc} \big|_{t \to \infty} = 4\alpha' \left\{ S^{(2)\mu\nu}_{(1)} \mathrm{Tr} \left( \frac{S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4l'}}{1 - S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4l'}} \right)^3 \right\} + S^{(1)\mu\nu}_{(1)} \mathrm{Tr} \left( \frac{S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4\tau}}{1 - S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4\tau}} \right)^3 \right\}, \tag{3.26}
\]

\[
\langle e^{ip \cdot X_{osc}} \rangle \big|_{t \to \infty} = \det \left[ \left( 1 - S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4l'} \right)^{-\frac{\alpha'}{2} p_{\mu} p_{\nu} S^{(2)\mu\nu}_{(1)}} \right] \times \det \left[ \left( 1 - S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4\tau} \right)^{-\frac{\alpha'}{2} p_{\mu} p_{\nu} S^{(1)\mu\nu}_{(1)}} \right] \\
\times \exp \left[ \frac{\alpha'}{2} p_{\mu} p_{\nu} S^{(2)\mu\nu}_{(1)} \mathrm{Tr} \left( 1 - S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4l'} \right)^{-1} \right] \\
\times \exp \left[ \frac{\alpha'}{2} p_{\mu} p_{\nu} S^{(1)\mu\nu}_{(1)} \mathrm{Tr} \left( 1 - S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4\tau} \right)^{-1} \right]. \tag{3.27}
\]

In fact, for obtaining Eq. (3.27) we assumed that the parameters of the setup and the momentum of the radiated closed string satisfy the condition

\[
p_{\mu} p_{\nu} \sum_{n=2}^{\infty} \left( S^{(1)*}_{(n)} + S^{(2)}_{(n)} \right)^{\mu\nu} = 0. \tag{3.28}
\]

This implies that for the given setup parameters, two components of the momentum of the radiated closed string are specified in terms of the other components.

Substitute Eqs. (3.24)-(3.27) into Eq. (3.17), the amplitude for the closed string radiation between the large distance branes is acquired.

### 4 The axion radiation

In this section we shall accomplish the amplitude of the Kalb-Ramond (axion) radiation from the interacting distant branes. The axion polarization tensor satisfies \( \epsilon_{\mu\nu} = -\epsilon_{\nu\mu} \).
and \( p^\mu \epsilon_{\mu \nu} = 0 \). Thus, Eq. (3.18) becomes

\[
\mathcal{M} = \epsilon_{\mu \nu} \left\{ 4 \alpha' \left( S^{(2)\mu \nu}_{(1)} K_{\nu} + S^{(1)\mu \nu}_{(1)} K_{\tau} \right) - \alpha'^2 \left[ p_\lambda p_\delta \left( S^{(2)\mu \lambda}_{(1)} S^{(2)\nu \delta}_{(1)} R_{\nu}^2 \right) - \left( S^{(2)\mu \lambda}_{(1)} S^{(1)\nu \delta}_{(1)} + S^{(1)\mu \lambda}_{(1)} S^{(2)\nu \delta}_{(1)} \right) R_{\tau} R_{\nu} + S^{(1)\mu \lambda}_{(1)} S^{(1)\nu \delta}_{(1)} R_{\tau}^2 \right) + k^\mu p_\lambda \left( S^{(2)\mu \lambda}_{(1)} R_{\nu} - S^{(1)\mu \lambda}_{(1)} R_{\tau} \right) + k^\nu p_\lambda \left( S^{(2)\mu \lambda}_{(1)} R_{\nu} - S^{(1)\mu \lambda}_{(1)} R_{\tau} \right) \right] \right\},
\]

(4.1)

where we have defined

\[ R_{(l', \tau)} = \text{Tr} \left[ \frac{S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4l' \tau}}{1 - S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4l' \tau}} \right] \left[ 1 + \frac{1}{1 - S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4l' \tau}} \right], \]

\[ K_{(l', \tau)} = \text{Tr} \left[ \frac{S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4l' \tau}}{\left( 1 - S^{(1)\dagger}_{(1)} S^{(2)}_{(1)} e^{-4l' \tau} \right)^3} \right]. \]

(4.2)

For comparing our results with the Ref. [34] we can express \( \mathcal{M} \) only in terms of the functions \( R_{(l', \tau)} \). For this purpose we impose the following condition on the setup parameters \( \Delta^{(1)\dagger}_{(1)} \Delta^{(2)}_{(1)} = 1 \). Afterward, for large distance limit the two-derivative term in Eq. (3.18) can then be written as

\[
\langle \partial X^\mu \partial X^\nu \rangle_{\text{osc}} |_{t \rightarrow \infty} = -\langle \partial X^\mu X^\nu \rangle_{\text{osc}} |_{t \rightarrow \infty} \left[ \langle p \cdot \partial X p \cdot X \rangle_{\text{osc}} |_{t \rightarrow \infty} + \frac{i \alpha'}{2} (k^2 - k'^2) \right]. \]

(4.3)

Exerting Eq. (3.25) into the one-derivative terms of Eq. (4.3), we receive Eq. (3.18) only in terms of the functions \( R_{(l', \tau)} \big|_{\Delta^{(1)\dagger}_{(1)} \Delta^{(2)}_{(1)} = 1} \). Besides, we have the following identity [34, 35]

\[
0 = \int_0^\infty d\tau \partial_\tau \left[ e^{-\tau (\alpha' k^2 - 4)} \exp \left( -\frac{1}{2} \alpha' p_\mu p_\nu S^{(1)\dagger \mu \nu}_{(1)} \text{Tr} \left[ \ln(1 - e^{-4\tau}) - (1 - e^{-4\tau})^{-1} \right] \right) \right] \]

\[
= \int_0^\infty d\tau \left[ \alpha' k^2 - 4 + 2 \alpha' p_\mu p_\nu S^{(1)\dagger \mu \nu}_{(1)} \tilde{R}_\tau \right] e^{-\tau (\alpha' k^2 - 4)}
\]

\[
\times \exp \left( -\frac{\alpha' p_\mu p_\nu S^{(1)\dagger \mu \nu}_{(1)}}{2} \text{Tr} \left[ \ln(1 - e^{-4\tau}) - (1 - e^{-4\tau})^{-1} \right] \right) \].
\]

(4.4)

Similar identity can be obtained for the integration over \( l' \) with \( \tilde{R}_l \), where

\[
\tilde{R}_\tau = -\frac{\alpha' k^2 - 4}{2 \alpha' p_\mu p_\nu S^{(1)\dagger \mu \nu}_{(1)}},
\]

\[
\tilde{R}_{l'} = -\frac{\alpha' k^2 - 4}{2 \alpha' p_\mu p_\nu S^{(2)\mu \nu}_{(1)}}.
\]

(4.5)
Now we acquired $\mathcal{M}$ in terms of the functions $\tilde{R}_\tau$ and $\tilde{R}_l$.

For receiving the final form of the axion amplitude we should use the following integral too

$$
\int_0^\infty d\tau \int_0^\infty d\tau' e^{-\tau(\alpha'k^2-4)} e^{-\tau'(\alpha'k'^2-4)} \langle e^{i p \cdot X_{osc}} \rangle |_{\Delta^{(1)}_{\alpha,\beta,\gamma,\lambda,\eta}} = 1 \approx \frac{1}{(\alpha'k^2-4)(\alpha'k'^2-4)},
$$

(4.6)

where for the correlator of the exponential we used the following form

$$
\langle e^{i p \cdot X_{osc}} \rangle |_{\Delta^{(1)}_{\alpha,\beta,\gamma,\lambda,\eta}} \approx \exp \left\{ -\alpha' p_\mu p_{\mu'} S^{(2)\mu\nu}_{(1)} \text{Tr} \left[ \ln(1 - e^{-4\tau'}) - (1 - e^{-4\tau'})^{-1} \right] \right\} \times \exp \left\{ -\alpha' p_\mu p_{\mu'} S^{(2)\mu\nu}_{(1)} \text{Tr} \left[ \ln(1 - e^{-4\tau}) - (1 - e^{-4\tau})^{-1} \right] \right\}.
$$

Let us apply the constant shifts on the momenta, i.e. $K^\alpha = k^\alpha + l^\alpha$ and $K'^\alpha = k'^\alpha + l'^\alpha$, in which the internal vector $l^\alpha$ satisfies the following conditions

$$
k.l = k'.l = 0, \quad l^2 = -\frac{4}{\alpha'}. \tag{4.8}
$$

Adding all these together we obtain the ultimate feature of the amplitude as in the following

$$
\mathcal{A} = \frac{1}{4} (2\pi)^{26} T_p^2 \mathcal{D}(y_1, y_2) \prod_{n=1}^{\infty} \left[ \det \left( Q^\dagger_{(n)} Q_{(n)2} \right) \right]^{-1}
\times \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \prod_{\tau=0}^{p} dK^\alpha dK'^\alpha \mathcal{D}(K; K') \epsilon_{\alpha\beta} \left( \frac{\Gamma_{\alpha\beta}}{K^2} - \frac{\Upsilon_{\alpha\beta}}{K'^2} \right),
$$

(4.9)

where $\Gamma_{\alpha\beta}$ and $\Upsilon_{\alpha\beta}$ are given by

$$
\Gamma_{\alpha\beta} = \frac{1}{p_{\mu} p_{\nu} S^{(1)\mu\nu}_{(1)}} \left[ p_\gamma \left( (K - l)^\alpha \Delta^{(1)\beta\gamma}_{(1)} + (K - l)^\beta \Delta^{(1)\gamma\alpha}_{(1)} \right) - \frac{1}{2} (K^2 - K'^2) \Delta^{(1)\alpha\beta}_{(1)} \right] - \frac{K'^2}{\left( p_{\mu} p_{\nu} S^{(1)\mu\nu}_{(1)} \right)^2} \left[ p_\gamma p_\eta \left( \Delta^{(1)\alpha\gamma}_{(1)} \Delta^{(1)\beta\eta}_{(1)} - \Delta^{(1)\alpha\beta}_{(1)} \Delta^{(1)\gamma\eta}_{(1)} \right) + p^\lambda p_\lambda \Delta^{(1)\alpha\beta}_{(1)} \right],
$$

$$
\Upsilon_{\alpha\beta} = \frac{1}{p_{\mu} p_{\nu} S^{(2)\mu\nu}_{(1)}} \left[ p_\gamma \left( (K - l)^\alpha \Delta^{(2)\beta\gamma}_{(1)} + (K - l)^\beta \Delta^{(2)\gamma\alpha}_{(1)} \right) - \frac{1}{2} (K^2 - K'^2) \Delta^{(2)\alpha\beta}_{(1)} \right] + \frac{K'^2}{\left( p_{\mu} p_{\nu} S^{(2)\mu\nu}_{(1)} \right)^2} \left[ p_\gamma p_\eta \left( \Delta^{(2)\alpha\gamma}_{(1)} \Delta^{(2)\beta\eta}_{(1)} - \Delta^{(2)\alpha\beta}_{(1)} \Delta^{(2)\gamma\eta}_{(1)} \right) - p^\lambda p_\lambda \Delta^{(2)\alpha\beta}_{(1)} \right]. \tag{4.10}
$$

The indices $\alpha, \beta, \gamma, \lambda, \eta$ belong to the set $\{0, 1, \cdots, p\}$ while $\mu, \nu$ are the spacetime indices. Since our calculations were in the lowest order Eq. (4.9) accurately is in the tree-level diagram.
The amplitude \((4.9)\) manifestly represents the axion radiation from the interaction of two parallel dynamical-dressed unstable Dp-branes in the large distance. As we see, according to the upper index \((1)\) \((2)\) the tensor \(\Gamma^{\alpha\beta}\) \((\Upsilon^{\alpha\beta})\) depends on the parameters of the first (second) brane. Therefore, the first and second terms of \((4.9)\) elaborate the axion radiation by the first and the second brane, respectively. According to the factors in Eq. \((4.9)\), a massless closed string state is emitted by one of the branes, then it is absorbed by the other brane, afterward it travels as an excited state on the brane for a while, it finally decays by radiating an axion state. This radiation is a bremsstrahlung-like process. We should mention that our results completely are in accordance with the Ref. \([34]\).

Note that in Eq. \((4.9)\) the term \(1/K^2K'\) is absent, which clarifies that there is no axion radiation near the middle points between the branes. For the distant branes the middle region is far from the both branes.

In fact, in some physical quantities the squared form of the amplitude, i.e. \(|A|^2\), is appeared. Let us call the amplitude \((3.1)\) as \(A_{12}\). By exchanging the branes we receive \(A_{21}\). We observe that the squared versions of them are not equal, i.e., \(|A_{12}|^2 \neq |A_{21}|^2\). This asymmetry precisely is due to the presence of the vertex operator of the radiated closed string. In other words, in the absence of any closed string radiation we acquire \(|A_{12}|^2 = |A_{21}|^2\). In this case we have only the pure interaction between the branes. The amplitude of the pure interaction is given by Eq. \((3.1)\) without the vertex operator and the \(\tau\)-integration. Note that even for the pure interaction there is \(A_{12} \neq A_{21}\). Therefore, the amplitude (for the pure interaction and / or radiation) is exactly elaborated by \(A_{12}\), but not with the arithmetic mean \((A_{12} + A_{21})/2\), e.g. see the Refs. \([3]\)-\([27]\), \([34]\).

5 Conclusions

We introduced a boundary state which is corresponding to a dynamical Dp-brane in the presence of the antisymmetric tensor field, an internal \(U(1)\) gauge potential with constant field strength and a specific open string tachyon field. For computing the amplitude of closed string radiation between two parallel Dp-branes, we combined the associated vertex operator of the radiated string and the boundary state formalism. We acquired the
radiation amplitude for a general massless closed string. Besides the arbitrary distance, the amplitude for large distances of the branes was also calculated. Presence of the various parameters drastically generalized the amplitude. By varying the parameters the value of the radiation amplitude can be accurately adjusted to any desirable value.

From the radiation amplitude of a general massless state we explicitly computed the axion production between the large distance $D_p$-branes. We observed that this radiation occurs only from one of the branes. Finally, we demonstrated that the axion production cannot occur near the middle region between the branes. This is due to the fact that the middle points are far from the both branes.

References


